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Plane hydrodynamic flow in thin free suspended nematic liquid crystal films

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Radial hydrodynamic flows in free suspended films (with thickness $h \geqslant 20\,\mu$m) of some liquid crystal materials was observed in a narrow temperature range before the line nematic–smectic C transition. The observed flows are explained by a non-linear temperature dependence of the surface tension (Marangoni effect).

Upon thinning the liquid crystal films to a thickness $6\,\mu\text{m} < h < 20\,\mu\text{m}$ the hydrodynamic flow changes its character: instead of radial flow with sinking seeding particles we observe two circular plane hydrodynamic flows symmetrical to the film's diameter. The temperature distribution in the free film and the surface tension field are discussed. A model for the established circular flows in thin liquid crystal films is presented.

A radial hydrodynamic flow in free suspended films with thicknesses $h \cong 40$–$50\,\mu$m of some liquid crystalline materials (OOBA, DOBA, HOBA and HOAB) was observed in a narrow temperature region ($\cong 5°$) before the nematic–smectic C phase transition [1, 2]. The observed flows are explained in terms of a non-linear behaviour of the free surface energy temperature dependence and by the so-called Marangoni effect [2, 3]. For these film thicknesses, two independent radial flows determined from two free surfaces are created, the surface flow direction is from cooler to warmer film parts, i.e. $d\sigma/dT > 0$ is valid for the free surface energy [2]. However, below certain thicknesses h ($6\,\mu\text{m} < h < 20\,\mu\text{m}$) the effects caused by nematic viscosity increase the correlation between the two free surfaces. Then at constant temperature in the indicated temperature intervals the hydrodynamic flow instability changes its character; it turns from radial into plane flow with two steady vortices, symmetrical to the film diameter (see figure 1). To determine the direction of the observed flows in some of the samples powder particles are especially included to avoid modifying the liquid crystal properties.

The free suspended films are obtained in the same way as in [1]. The films are spread on a cut out circular hole ($2R = 1$ mm) in the middle of a foil with size $20 \times 20 \times 0{\cdot}02$ mm and this foil is placed in a thermostat. The investigations are carried out with a polarizing microscope. By focusing on to defects on the two free surfaces the film thicknesses are determined. A radial temperature gradient about $3\,\text{K mm}^{-1}$ for films with $h > 20\,\mu$m is experimentally established by the earlier appearance of the S_C phase in the middle of the film at the N–S_C phase transition and the existence of a temperature irregularity $\Delta T_R < 0{\cdot}5$ K along the film periphery. The vertical temperature gradient can be neglected.

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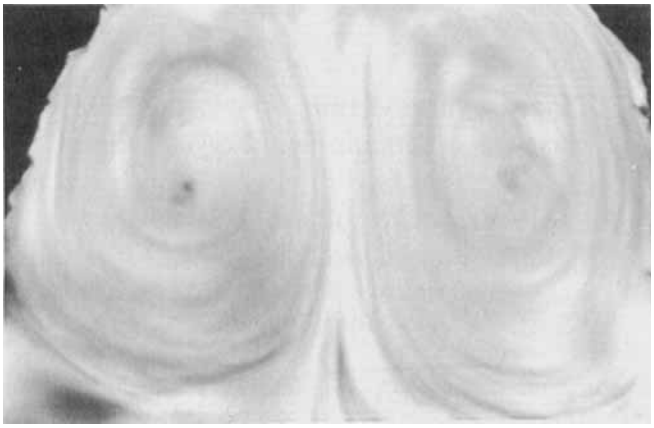

Figure 1. Photograph of the texture ($P \perp A$) of a freely suspended film of OOBA at 110°C with two vortices.

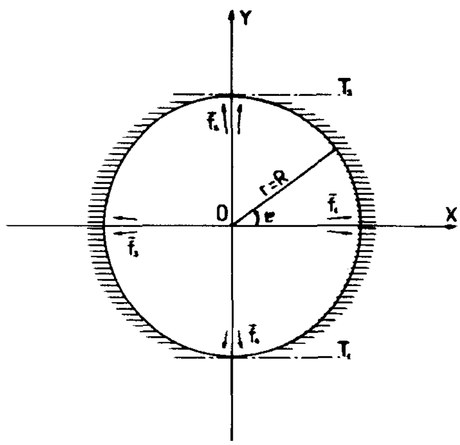

Figure 2. Scheme of the circular free film with the field of surface forces $\mathbf{f}_n$.

Vortex flow in thin films of 8OCB, 7OCB and EBBA, investigated under the same experimental conditions is not observed. This observation and the fact, that the hydrodynamic flow exists for the temperature interval $T_{NS_C} < T < T_{NS_C} + 5$ K and is absent in the rest of the nematic region for OOBA, DOBA, HOBA and HOAB substances allows us to neglect the diffusion and convective forces and to pay attention just to the action of capillary forces in the films.

Let us suppose (see figure 2) a linear variation of the foil temperature along the Y axis. Such variation is possible due to the existing configuration of thermostat heaters and the small size of the foil hole, so that $T_1 < T_2$. Then if we choose a set of polar coordinates with the origin placed in the film centre, we can describe the temperature distribution along the circular hole periphery in the following way:

$$T_R = T_R(\varphi = 0) + a \sin \varphi.$$

Let us try to find the temperature distribution for an immobile circular film in which there is heat exchange on its free surfaces and heat sources along its periphery with intensity per unit length

$$q = q_0 + q_1 \sin\varphi,$$

where q_0 and q_1 are constants. The surface film bending is neglected.

The heat conduction equation for films with thickness h and temperature $T(r, \varphi)$ in polar coordinates is:

$$h\alpha^2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}\right) - 2\beta(T - T_1) = 0,$$

where α^2 is a heat conduction constant, β is a constant of heat exchange with outside space, T_1 is also a constant, i.e. the environment temperature. The solution of this equation with the above boundary conditions is

$$T = \frac{R}{h\alpha^2}[q_0 K_0(AR) I_0(Ar) + q_1 K_1(AR) I_1(Ar) \sin\varphi + T_1], \tag{1}$$

where $A = 2\beta/h\alpha^2$; K_0, K_1, I_0 and I_1 are the modified Bessel functions [4].

The film temperature distribution in equation (1) defines the existing thermocapillary surface tensions which are described by the free surface energy gradient σ:

$$\operatorname{grad}\sigma = \frac{\partial \sigma}{\partial r}\mathbf{e}_r + \frac{\partial \sigma}{\partial \varphi}\mathbf{e}_\varphi, \tag{2}$$

where $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ are unit vectors. It is well-known that if the free surface energy gradient is present, then the effective tangential force is directed to the region with higher free surface energy. Then for the components of grad σ we obtain from equations (1) and (2) [5]:

$$\begin{aligned}\frac{\partial \sigma}{\partial r} &= \frac{d\sigma}{dT}\frac{\partial T}{\partial r} = \frac{d\sigma}{dT} A[q_0 K_0(AR) I_1(Ar) + \tfrac{1}{2} q_1 K_1(AR)[I_0(Ar) + I_0(Ar)] \sin\varphi] \\ \frac{\partial \sigma}{\partial \varphi} &= \frac{d\sigma}{dT}\frac{\partial T}{\partial \varphi} = \frac{d\sigma}{dT} q_1 K_1(AR) I_1(Ar) \cos\varphi.\end{aligned} \tag{3}$$

Let us investigate the term $\partial\sigma/\partial\varphi$. Its average value in the interval $[0, 2\pi]$ when r is a constant is equal to zero. This means, that a circular motion with radius r in the film plane cannot be expected. The other component $\partial\sigma/\partial r$ determines a radial film tension, which is formally illustrated by means of local vectors $\mathbf{f}_n = (\partial\sigma/\partial r)\mathbf{e}_r$ in figure 2, and according to equation (3) they are $|\mathbf{f}_4| < |\mathbf{f}_1| < |\mathbf{f}_2|$ and $|\mathbf{f}_1| = |\mathbf{f}_3|$. From the qualitative presentation of the free surface energy gradient we can make the following conclusion: if the thermocapillary tension reaches sufficient values to overcome the viscosity force, a hydrodynamic instability along the y axis can occur. Since there are no sources and collectors of mass in the film, hydrodynamic instability can have just a vortex character. Let us suppose a presence of vortex flow in the film. By means of a current function ψ (r, φ) we can write for a vortex vector ω

$$\omega = -\nabla^2\psi. \tag{4}$$

For two dimensional flow $\omega = f(\psi)$ [6]. We will investigate equation (4) for the case when $f(\psi)$ is a linear function, i.e. $f(\psi) = k^2\psi$ where k is a constant or

$$\omega = k^2\psi. \tag{5}$$

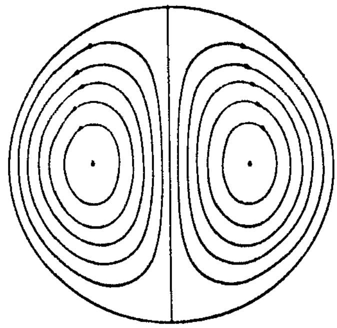

Figure 3. Theoretical solution of the vortex problem for freely suspended nematic films.

Then equation (4) can be rewritten for ψ in polar coordinates as

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2}=-k^2\psi. \tag{6}$$

A solution of type $\psi \propto \sin\varphi$ is possible, and a general solution of equation (6) in this case is

$$\psi = CJ_1(kr)\sin\varphi, \tag{7}$$

where $J_1(kr)$ is the modified Bessel function and C is a constant. To equation (7) we impose the boundary condition $\omega=0$ when $r=R$. Then from equations (5) and (7) there follows the requirement

$$J_1(kR)=0. \tag{8}$$

The least possible value of kR, which satisfies equation (8) is $kR=3{\cdot}83$ [6]. For this the pattern mode of current lines in the region $r \leqslant R$ is presented in figure 3. The comparison of the obtained solution (see figure 3) with the experimental results (see figure 1) confirms the substitution $f(\psi)=k^2\psi$. Sometimes the experimental picture shows some deviation with respect to the theoretical, i.e. an asymmetry of vortexes towards the X axis is observed. This fact can be explained with an irregular thermocapillary tension distribution along the y axis. The vortex textures observed at the N–I phase transition show a displacement of film minimum temperature, which according to equation (1) is in the film centre, to the warmer film part. This is due to the infusion of cooler hydrodynamic mass flow.

We point out again, that the plane hydrodynamic flow in the film is observed for the thickness interval ($6\,\mu m < h < 20\,\mu m$). For thinner films a new behaviour of type $d\sigma/dT$ and a lack of hydrodynamic flow is observed. This case will be treated in a forthcoming paper.

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